Slope Stability

Alessio FERRARI (AF)

Exercise 3 - Solution

LIMIT EQUILIBRIUM ANALYSES – SIMPLIFIED BISHOP METHOD

Part 1

In Figure 1 the drawing of a dry slope (sandy silt) is given. A circular slip surface delimited by points A and B is assigned. The origin $O \equiv (0;0)$ of the reference system is chosen to be at the slope toe (point A). Geometry and soil properties are provided in Table 1.

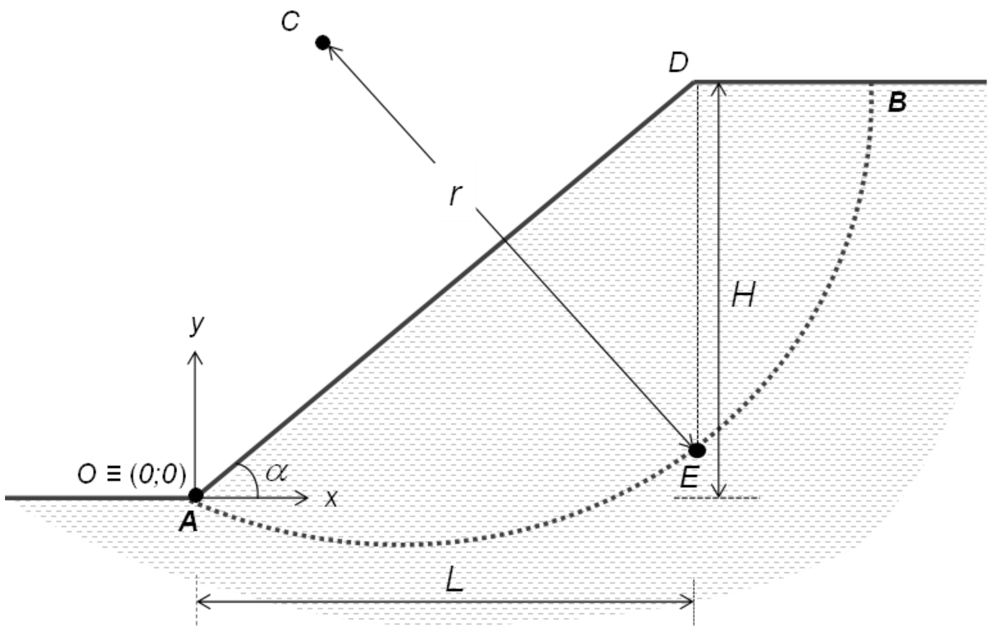

Figure 1 slope of gem slope geometry

Table 1: geometry and soil properties of the slope given in Figure 1.

γ_{sat} (kN/m^3)	w_{sat} (%)	α (°)	H (m)	L (m)	x_C (m)	y_C (m)	r (m)	φ' (°)	c' (kPa)
21.0	16.7	30.0	8.0	13.9	7.0	10.0	12.2	22.0	5.0

After dividing the analysis domain in n=20 slices, compute F for the assigned slip surface according to the simplified Bishop's method.

Solution

This solution adopts a domain division such that $\Delta x_i = (L + \text{DB}) / 20 = 19.03\ \text{m} / 20 = 0.95\ \text{m}$ is the constant width for each slice. DB length is evaluated after the computation of coordinates x and y of B knowing that the equation of the circular slip surface is $(x-x_c)^2 + (y-y_c)^2 = R^2$.
In order to compute the weight W_i for each slice, γ_d has to be computed: $\gamma_d = \gamma_{sat} / (1+w) = 18.0\ \text{kN/m}^3$.
With reference to a thickness in 3rd direction of 1 m, $W_i = \gamma_d A_i$. Table 2 lists the weights of slices:

Table 2: area and weight for each slice n_i according to the division keeping Δx_i constant. Weight is computed considering 1 m thick slice.

n (-)	A_i (m^2)	W_i (kN)
1	0.55	9.87
2	1.59	28.63
3	2.53	45.57
4	3.38	60.89
5	4.15	74.74
6	4.84	87.20
7	5.46	98.35
8	6.01	108.22
9	6.49	116.80
10	6.89	124.10
11	7.23	130.07
12	7.48	134.63
13	7.65	137.68
14	7.73	139.06
15	7.65	137.77
16	7.05	126.92
17	6.22	111.89
18	5.17	92.97
19	3.77	67.87
20	1.50	27.02

The procedure for the computation of the safety factor according to the simplified Bishop's method is below summarized:

Rotational equilibrium equation:

$$F = \frac{\sum_i T_i r}{\sum_i W_i r sin\alpha_i} = \frac{\sum_i (c'\Delta l_i + N'_i tan\phi')}{\sum_i W_i \, sin\alpha_i}$$

Equilibrium of the slice in the vertical direction:

$$N'_i = \frac{W_i - (X_i - X_{i-1}) - U_{bi} cos\alpha_i - \frac{c'\Delta l_i sin\alpha_i}{F}}{cos\alpha_i + \frac{sin\alpha_i tan\phi'}{F}}$$

or:

$$N'_i = \frac{W_i - (X_i - X_{i-1}) - U_{bi} cos\alpha_i - \frac{c'\Delta l_i sin\alpha_i}{F}}{m_{\alpha_i}}$$

with

$$m_{\alpha_i} = cos\alpha_i + \frac{sin\alpha_i tan\phi'}{F}$$

By assuming that for each slice the tangential inter-slice forces have equal magnitude and opposite direction, the term $(X_i - X_{i-1})$ is equal to 0.

By substituting the expression of N'_i in the rotational equilibrium equation, one can get:

$$F = \frac{\sum_i T_i r}{\sum_i W_i r sin\alpha_i} = \frac{\sum_i \left[c'\Delta l_i + \left(\frac{W_i - U_{bi} cos\alpha_i - \frac{c'\Delta l_i sin\alpha_i}{F}}{m_{\alpha_i}} \right) tan\phi' \right]}{\sum_i W_i \, sin\alpha_i}$$

$$F = \frac{\sum_i \left[\frac{c'\Delta l_i m_{\alpha_i} + \left(W_i - U_{bi} cos\alpha_i - \frac{c'\Delta l_i sin\alpha_i}{F} \right) tan\phi'}{m_{\alpha_i}} \right]}{\sum_i W_i \, sin\alpha_i}$$

$$F = \frac{\sum_i \left[\frac{\left(c'\Delta l_i m_{\alpha_i} - \frac{c'\Delta l_i sin\alpha_i}{F} tan\phi' \right) + (W_i - U_{bi} cos\alpha_i) tan\phi'}{m_{\alpha_i}} \right]}{\sum_i W_i \, sin\alpha_i}$$

$$F = \frac{\sum_i \left\{ \frac{\left[c'\Delta l_i \left(cos\alpha_i + \frac{sin\alpha_i tan\phi'}{F} \right) - \frac{c'\Delta l_i sin\alpha_i}{F} tan\phi' \right] + (W_i - U_{bi} cos\alpha_i) tan\phi'}{m_{\alpha_i}} \right\}}{\sum_i W_i \, sin\alpha_i}$$

$$F = \frac{\sum_i \left[\frac{\left(c' \Delta l_i cos\alpha_i + \frac{c' \Delta l_i sin\alpha_i tan\phi'}{F} - \frac{c' \Delta l_i sin\alpha_i}{F} tan\phi' \right) + (W_i - U_{bi} cos\alpha_i) tan\phi'}{m_{\alpha_i}} \right]}{\sum_i W_i \, sin\alpha_i}$$

It follows :

$$F = \frac{\sum_i \left[\frac{(c' \Delta l_i cos\alpha_i) + (W_i - U_{bi} cos\alpha_i) tan\phi'}{m_{\alpha_i}} \right]}{\sum_i W_i \, sin\alpha_i}$$

or :

$$F = \frac{\sum_i \left[\frac{c' b_i + (W_i - U_{bi} cos\alpha_i) tan\phi'}{m_{\alpha_i}} \right]}{\sum_i W_i \, sin\alpha_i}$$

with :

$$b_i = \Delta l_i cos\alpha_i$$

$$m_{\alpha_i} = cos\alpha_i + \frac{sin\alpha_i tan\phi'}{F}$$

The safety factor *F* is unknown, and has to be achieved by an iterative procedure. The analysis can be implemented in an Excel spreadsheet. The *Goal Seek* tool can be used. Three main steps have to be followed:

- writing in a cell a first attempt safety factor ($F_{hypothesis}$);
- writing in a cell the expression of the safety factor according to Bishop's simplified method as function of the first attempt safety factor F = f($F_{hypothesis}$);
- minimization of the difference F-$F_{hypothesis}$ setting it equal to zero by changing $F_{hypothesis}$.

The results in terms of $N_i^{'}$, $T_{i,f}$ and T_i are given in Table 3. The safety factor for dry condition results to be **F = 1.71**.

Table 3: N_i, $T_{i,f}$ and Ti for each slice n_i.

n (-)	$N_i^{'}$ (kN)	$T_{i,f}$ (kN)	T_i (kN)
1	16.19	12.17	-5.28
2	38.46	20.89	-13.08
3	55.87	27.72	-17.26
4	69.98	33.26	-18.32
5	81.72	37.90	-16.65

6	91.71	41.86	-12.62
7	100.34	45.31	-6.56
8	107.90	48.35	1.23
9	114.60	51.08	10.44
10	120.57	53.54	20.77
11	125.92	55.78	31.92
12	130.72	57.84	43.55
13	135.02	59.75	55.28
14	138.83	61.51	66.69
15	141.31	62.83	76.84
16	135.18	60.78	80.71
17	125.55	57.52	79.91
18	112.14	53.11	73.69
19	90.43	46.25	59.16
20	34.60	30.07	25.81

Part 2

Compute F by considering the case of submerged slope (Figure 2). Discuss briefly the result by comparing it to the one achieved in the previous case.

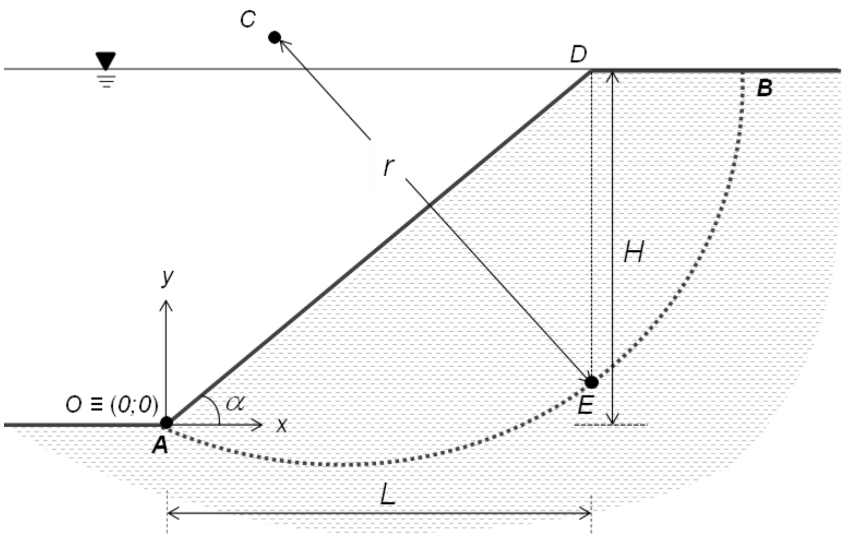

Figure 2 slope geometry in submerged conditions

After dividing the analysis domain in n=20 slices, compute F for the assigned slip surface according to the simplified Bishop's method.

Solution

The condition of submerged slope implies that water pressure is hydrostatically distributed for each slice. The force resulting from the water pressure distribution is equivalent to the buoyant force. In this sense the system is statically equivalent to a system in which the unique force is the gravitational one, computed with a unit weight equal to buoyant one ($\gamma' = \gamma_{sat} - \gamma_w = 11.2$ kN/m^3). The procedure implemented for the previous case is still valid for the computation of the safety factor if the buoyant unit weight (γ') is used instead of the dry unit weight (γ_d).

Table 4: buoyant weight W_i' for each slice n_i.

n (-)	W_i' (kN)
1	6.03
2	17.50
3	27.85
4	37.21
5	45.67
6	53.29
7	60.10
8	66.13
9	71.38
10	75.84
11	79.48
12	82.27
13	84.14
14	84.98
15	83.76
16	77.56
17	68.38
18	56.82
19	41.47
20	16.51

The safety factor for the submerged condition results to be **F = 1.85**.

Table 5: N_i, $T_{i,f}$ and T_i for each slice n_i.

n (-)	N_i' (kN)	$T_{i,f}$ (kN)	T_i (kN)
1	6.60	9.88	-3.23
2	20.50	14.97	-8.00
3	34.89	18.99	-10.55
4	43.57	22.29	-11.19
5	50.82	25.06	-10.17
6	57.01	27.44	-7.71
7	62.37	29.54	-4.01
8	67.07	31.40	0.75
9	71.24	33.09	6.38
10	74.97	34.63	12.69
11	78.32	36.05	19.51
12	81.33	37.37	26.61
13	84.02	38.62	33.78
14	86.40	39.81	40.76
15	87.95	40.56	46.71
16	84.05	39.65	49.32
17	77.88	37.85	48.83
18	69.18	35.42	45.03
19	54.80	31.64	36.16
20	15.81	22.68	15.77